What Every Game Designer Should Know About Game Theory

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Outline

- Game theory myths
- Basic concepts
- Risk
- Repeated games & discounting
- Knowledge & Belief
- Practical examples
- Conclusions

Myth 1: Game Theory Only Predicts What Game Theorists Do

- Origins
  - MAD
  - Game Theory Students
- Facts
  - Repeated games useful
  - Quantitative politics
    - Bruce Bueno de Mesquita
  - Protect against worst case

Myth 2: Game Theory is for Eggheads, Game Design is an Art

- Origin
  - Traditional game design
  - Bad models worse than none
- Facts
  - The math & science is now here
    - Finance (post 1970), politics (now), behavioral econ (achievements)
  - Save $ in testing, player satisfaction???

Myth 3: Solving Games Is Hard

- Origins
  - Finding equilibrium is NP-Hard (exponential)
- Facts
  - Game designers are designers, not players
    - Solve upfront
  - Can model abstract version
    - Heuristics
  - Often structure in data (e.g., Sandholm, AIJ, '02)

Myth 4: Too Many Solutions

- Origins
  - Uncountable & infinite number of equilibria
  - Doesn't predict which one
- Fact
  - Good for games!

Skill vs. Strategy

- Skill
  - Driven by capabilities, signaling, reputation
  - Measured using statistics, hindsight
- Strategy
  - Driven by preferences (valuations), sanctioning, trust
  - Solved using game theory, foresight
- Bounded rationality
  - Agency: tic-tac-toe vs sudoku vs chess
  - Solve game → skill: winner/draw/random

Desiderata

- Nash equilibrium (NE): optimal strategy given circumstances
  - Evolutionary Stable Strategy (ESS): Subset of NEs
- Pareto frontier: improve with none worse off
- Not always coincide
  - “Mexican Standoff”

NASCAR

Ummm...


Wii
NASCAR
KART RACING
EVERYONE
E
CONTENT RATED BY
ESRB
freestyle

NASCAR: Drafter's Dilemma

	Cooperate	Defect
Cooperate	3 3	-5 3
Defect	2 -5	1 1

- Red ahead, Blue behind, leave line together
- Payoff = number of cars passed
- Cooperate = allow other to jump back in line
- Defect = jump back in line without the other

Ronfeldt, First Monday J., '00

Dominant Strategy & Risk

	Stag		Hare	
Stag	10	10	0	8
Hare	8	0	7	7

- Nash equilibrium
- Payoff dominance vs risk dominance
- Cooperation

Risk

- Expected Utility = Σ probability * utility
  - Quasilinearity
- Risk averse/neutral/seeking
- Save points, powerups/items, loss

Risk & Commitment
Game of Chicken

	Swerve	Straight
Swerve	0 0	-1 +1
Straight	+1 -1	-1000 -1000

- Credible threats
  - Deliberately limit freedom
  - Leave opponent exit
  - Bluffs

Mixed Strategy & Risk

			
	0, 0	-1, 1	1, -1
	1, -1	0, 0	-1, 1
	-1, 1	1,-1	0, 0

Street Fighter 4

- Intransitivity
- "Every unit overpowered"
- Forced risk

Repeated Games: Skill & Intransitivity

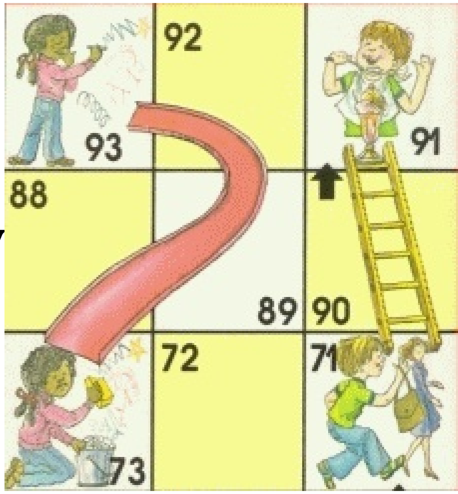
- Voting/Ranking intransitivity
  - A > B > C,   B > C > A,   C > A > B
- Eigenvector centrality methods
  - Kiss-the-moose: the traveling wood chip
  - Relative weight & importance
  - Logarithmic variation used in NCAA
  - Google

from www.cowart.info

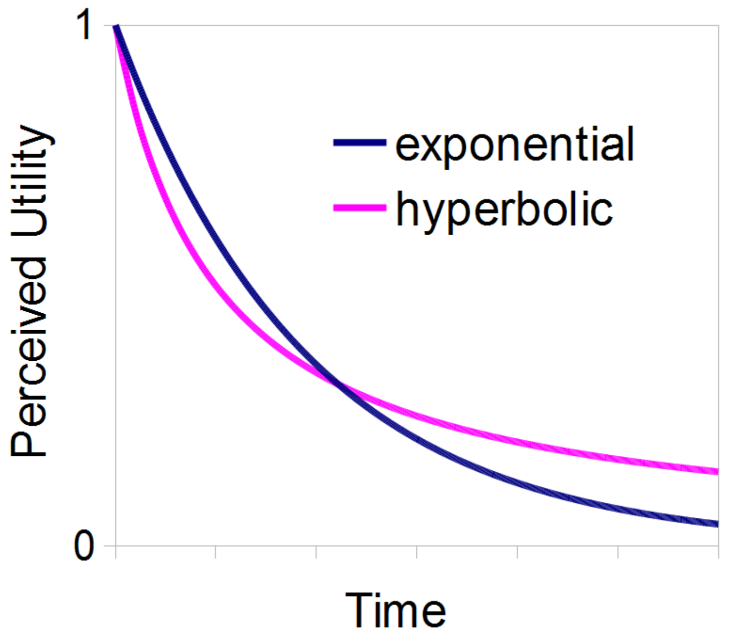
Repeated Games: Ergodicity

- Nonergodic: transient states & sinks
  - Backward induction
- Practically ergodic (.001% chance of return) vs mathematically ergodic
  - Unreachable gameplay
  - Martingale processes
- People not always follow ergodicity
  - Habits
  - Assumptions/Knowledge
- Open systems: evolutionary stable strategies

Chutes and Ladders



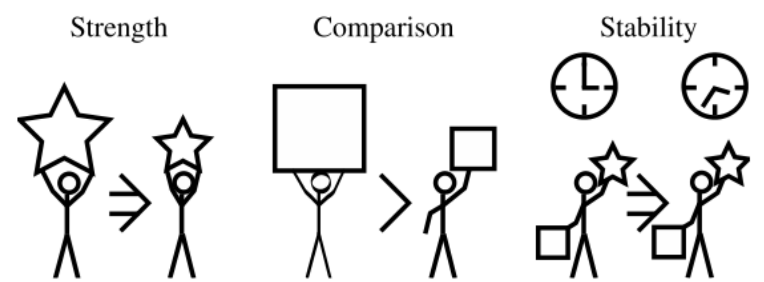
Discounting



- Uncertain future
  - Affect of delay on reward
  - Influenced by: patience, beliefs, risks, exogenous discount factors & value
- Expected utility =
  - Exponential, dynamically consistent: $\Sigma\ \gamma^t\ u$
  - Hyperbolic, realistic hazard rate: $\Sigma\ 1/(1+\gamma t)\ u$

Discounting In Repeated Interactions

- patience = discount factor
  discount factor + utilities = trustworthiness
  (Hazard & Singh, TKDE, '10)
- Dictates reciprocity
  (Hazard, COIN, '08)



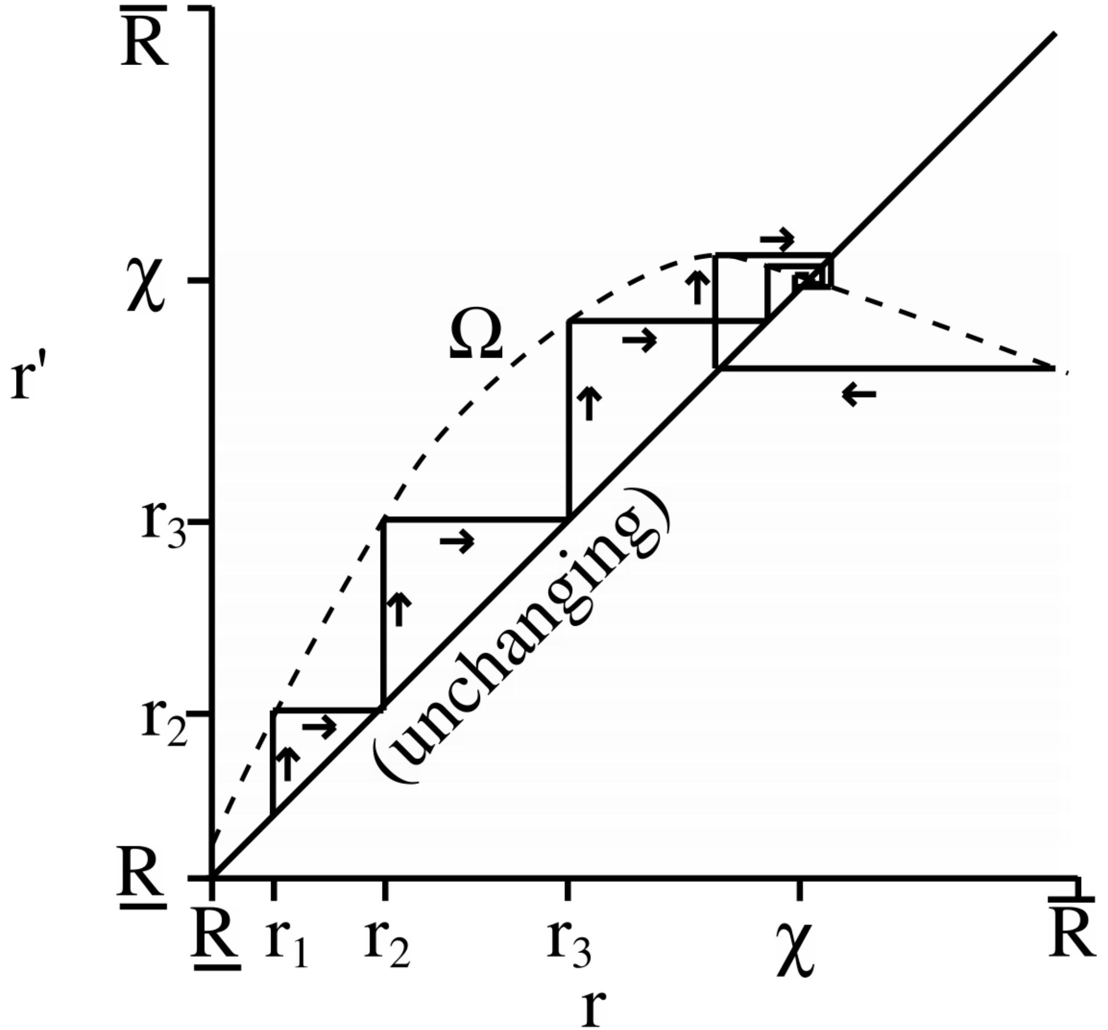
- Risk perception
  - Temporal pressure good: pacing vs caution
  - Temporal pressure bad: frustration
- Amortize costs over expected usefulness

Feedback

- Positive feedback (amplify)
  - Done right: separates skill & strategy
  - Too strong / early: random outcomes
- Negative feedback (dampen)
  - Done right: keeps game engaging
    - “Elastic Band”
  - Too late: prolongs inevitable, random outcomes
  - Too much:
    - Frustrate good players
    - perverse incentives (not always bad)

Feedback Analysis

Power Ratio r: Player 1 DPS/Player 2 DPS



A Simple Game...

- Strategist
- Negotiator
- Artist
- Logician (e.g., programmer/lawyer)
- Impulsivist or risk seeker
- Risk avoider

Rules

Card is cost:

A: 1

2: 2

3: 3

…

J: 11

Q: 12

K: 13

- Bid each round
- Winning bid gets price – cost
- Highest total wins

Knowledge & Belief

- Mutual information vs common knowledge
  - Did the message arrive?
  - Trust
  - Out-thinking
  - Mixed strategy: human ability to be random
    - Coin flips
- Communication
  - Low cost vs high cost
  - Aggregation
  - Costly lies only mitigated by strong sanctions

The Princess Bride

Keynesian Beauty Pageant: Guess 2/3 the average

- Everyone choose number [0,100]
- Closest to 2/3 the average wins
- Results
  - Rationality is common knowledge: 0
  - Human experiments: 20's typical
  - Fads & bads

Image from thedigeratilife.com

Exploration vs Exploitation

- Multiarmed bandit
  - Knowledge discovery
- Optimal point of trade-off
  - Discount factor
  - Opponents
  - Risk

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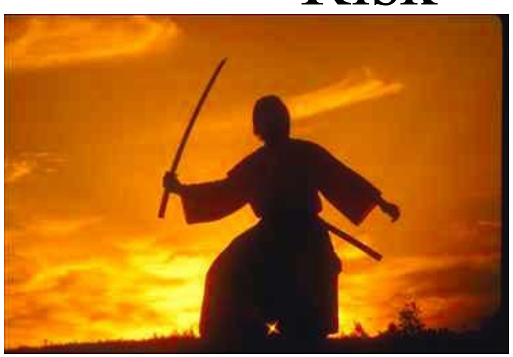
- Strategic concealment
  - Increase costs of discovery
  - Baggy clothes hide position, weapon



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War of Attrition

	Hawk	Dove
Hawk	-5 -5	10 0
Dove	0 10	3 3

- Both want resource, one gets it
  - Auction
  - Taking out the trash
  - Sniping (boring vs winning)
- Combines repeated games, belief, risk, discounting

Utility & Currency

- Common currency: **average-player time**
  - Skilled players & devoted players have most
- Find exchange rates for everything
  - If items purchasable in $, *find exchange between player time and $*
- Find amortization / discount rate

Player Encounter Model

- Game design structures
  - Poisson
  - Uniform
- Social structures
  - Self-similarity, power laws
  - Model using Kronecker products of edge matrices (Leskovec & Faloutsos, ICML, '07)

Pricing Example

- Weapons for sale:

Pricing Example

- Weapons for sale:
  – MC Hammer

Pricing Example

- Weapons for sale:
  - MC Hammer
  - Britney Spear (+5 Auto-Tune bonus)

Pricing Example

- Weapons for sale:
  - MC Hammer
  - Britney Spear (+5 Auto-Tune bonus)
  - Curse of the tax audit
    - Not immediate – need to discount the effects first

Model Components

- Input sets
  - S: matrix of relative weapon strengths
  - C: vector of weapon costs
    - Multiple currencies → average-player time
  - P: probability player will buy weapon in NE
- Constraints
  - At best, have full control over 2 input sets

Strength and Utility

S (strength: # of player 1 to defeat player 2)

	Hammer	Spear	Curse
Hammer	1	3	0.5
Spear	0.33	1	0.5
Curse	2	2	1

U (utility)

	Hammer	Spear	Curse
Hammer	0.000	-0.043	0.095
Spear	0.043	0.000	-0.070
Curse	-0.095	0.070	0.000

C (cost)

	Cost
Hammer	0.23
Spear	0.56
Curse	0.21

- One player loses all utility, another fraction
- Spear vs Hammer:

  gain - loss

  0.23 - (1/3 * 0.56)
- Symmetric!

Probability

U (utility)

	Hammer	Spear	Curse
Hammer	0.000	-0.043	0.095
Spear	0.043	0.000	-0.070
Curse	-0.095	0.070	0.000

P (probability)

	Probability
Hammer	0.336
Spear	0.456
Curse	0.208

P (probability)

	Probability
Hammer	0.333
Spear	0.334
Curse	0.333

U (utility)

	Hammer	Spear	Curse
Hammer	0.000	-0.073	0.073
Spear	0.073	0.000	-0.073
Curse	-0.073	0.073	0.000

C (cost)

	Cost
Hammer	0.255
Spear	0.545
Curse	0.200

For the Math Folks

Constraints:

$$||C||_1 = 1$$

$$||P||_1 = 1$$

$$s_{ij} = \begin{cases} 1 \text{ when } i = j \\ 1/s_{ji} \text{ otherwise} \end{cases}$$

$$u_{ij} = 1/\max(1, s_{ij}) * c_i - 1/\min(1, s_{ij}) * c_j$$

PU=**0**

Gotchas

- "All models are wrong, some are useful"
- Impossibility
  - Good, Fast, & Cheap
  - Economies: budget balanced, incentive compatible, individually rational, & efficient

    (Myerson & Satterthwaite, J. Econ Theory, '83)
  - Voting: no ideal system (Arrow, J. Political Econ., '50)
  - Revelation principal: honesty at what cost?
- Be careful with probability (e.g., Monte Hall problem)

Conclusions

- Game theory is useful for modeling people
- Game theory prevents griefs
- Make sure abstraction matches game
  - Integration with AI
- Don't forget repeated interactions
- **You can mathematically design the game you want players to play**