Coordination in multi-agent systems: The effects of economies of scale and switching costs

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Outline

Definitions and Motivation

Game of Scale Model

GoS Strategies

GoS Simulation Results

GoS Control & Design

Minority Game: No Switching Costs

Majority Game: Reversed Minority Game

Coordination

- Coordination: agents desire to agree on same or corresponding choices
- Anti-coordination: agents desire to agree with as few others as possible (e.g. congestion)

Economies of Scale

- Found in
  - Energy distribution & storage
  - Product compatibility
  - Electronic Services
    - Connectivity
    - Services/portals
    - Formats
- Often ignored in innovation diffusion, network externalities

Switching Costs

- Endowment & choice
- Examples
  - Purchasing a durable good
  - Implementing a protocol
  - Switching time

Coordination & Innovation Diffusion

- Epidemic learning, supply & demand, subsidies
  - Specific to econ
  - Stoneman et al. '86a, '86b, '86c, '03, '05
- Majority game
  - No switching cost, drastically changes model
  - Later in talk
- Punctuated equilibrium w/ linear cost
  - Only works with linear models
  - Loch & Huberman '99
- Congestion games
  - Diseconomies of scale
  - Blumrosen & Dobzinski '06

The Game of Scale

- Strategic behaviors for agents & system controller
- A game that expresses:
  - Economies of scale
  - Many agents
  - Low model dimensionality (simple)
  - Switching costs
- Joint work with Peter Wurman
  (formerly at NCSU, now at Kiva Systems)

Game of Scale Properties

- $N = \#$ of agents
- $n_i(t) = \#$ of agents using i at time t
- Non-decreasing cost function, e.g.
  - $c_i(n_i) = b_i + \frac{a_i}{n_i/N}$
  - $c_i(n_i) = b_i + a_i e^{-d_i \frac{n_i}{N}}$
- Switching cost distribution, e.g.
  - $S \sim N(\mu_s, s_s)$
  - $S \sim U(\mu_s - \frac{w_s}{2}, \mu_s + \frac{w_s}{2})$

Example

Time	Cost 0	Cost 1	Agent A ($S = 2$)	Agent B ($S = 4$)	Agent C ($S = 1$)
0	6	3	**0**	**0**	**1**
1	6	3	0: cost 6 **1**: cost 5	**0**: cost 6 1: cost 7	0: cost 7 **1**: cost 3
2	8	2	0: cost 10 **1**: cost 2	0: cost 8 **1**: cost 6	0: cost 9 **1**: cost 2

Simple Pure-Strategy Nash Equilibria

- Strictly dominant cost functions
- Oscillating innovations
  - $(T_j - T_i)c_h(1) > (T_j - T_i)c_i(N) + S_{\max}$ (discount factors permitting)
  - Oscillation of subset of agents
- Socially optimal NE may be payoff dominant but not risk dominant

Decision Models (Agent Types)

- Possibly risk averse agents
- Knowledge of switching cost distribution
- Types
  - Equilibrium
  - Myopic
  - Trend-Following
- Explore for 2 innovations

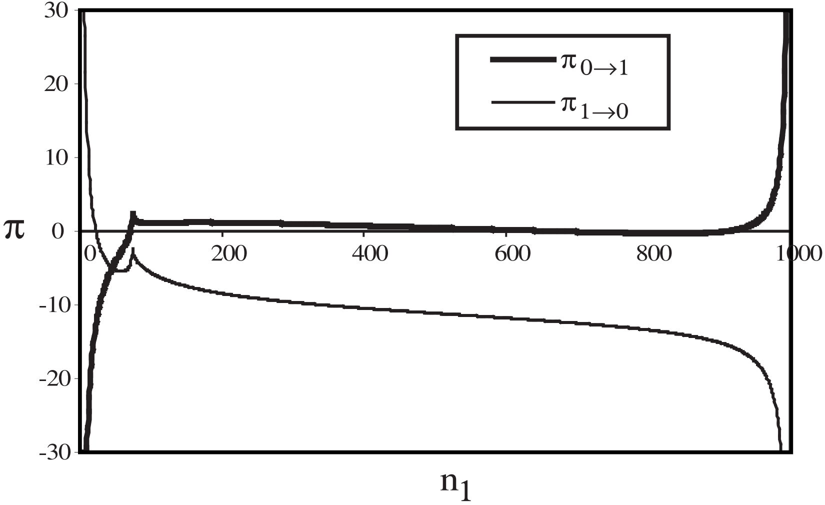
Equilibrium Agents

- Switch immediately to expected equilibrium
  - Non-repeated game
- Find equilibrium:
  - Find switching costs with indifference to switching
  - Assume other agents with cheap switching costs will switch
  - Switch if profitable based on switching cost quantile given expected equilibrium
- Now or never (e.g. high retooling costs)

Myopic Agents

- Low discount factor
- Switch from i to j only if
  $c_i(n_i) > c_j(n_j + 1) + S$
- Same behavior synchronous, Poisson, round-robin
- Wait-and-see

Myopic Agents' Profit



Trend-Following Agents

- Discounted Taylor series
  - Extrapolate current trends
  - Use discrete approx for derivatives
- Cost to switch at time k
  - Cost of switch + cost before & after

$$c_{i \to j}(k) = (1-\delta_{ij})\gamma^k S + \sum_{l=0}^{k-1} \gamma^l c_i(\tilde{n}_i(t+l)) + \sum_{l=k}^{\infty} \gamma^l c_j(\tilde{n}_j(t+l))$$

- Is **now** the best time to switch?

Trend-Following Agents (2)

- Can approximate convergence
  - $O(\gamma^{-l}) > O(c_i(l))$
- Clamp $\tilde{n}$ to $[1, N]$
- 3 derivatives is plenty
- Public Monitoring
  - Media
  - Hype

Dynamic Behavior

$N = 1000$

$n_1(0) = 74$

$\mu_s = 5.5$

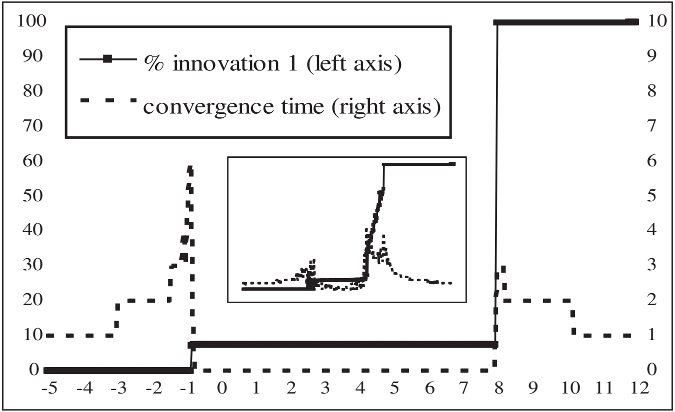
$b = 6$

$a_0 = 0.180$

$a_1 = 0.282$

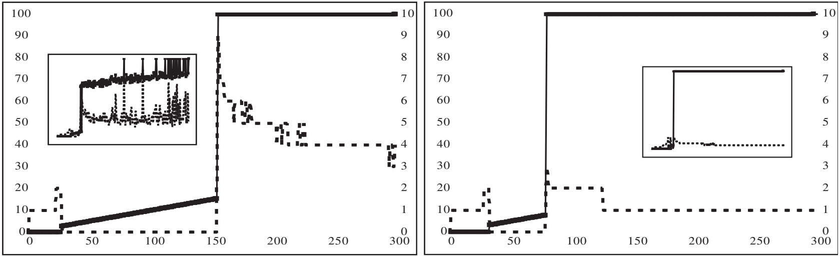
$w_s = 2.2$ (uniform)

$\sigma_s = 1.7$ (Gaussian)

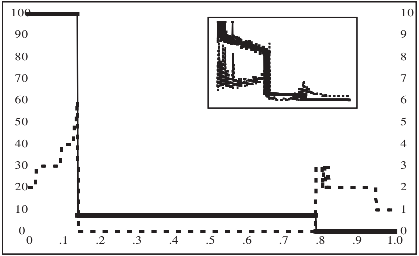


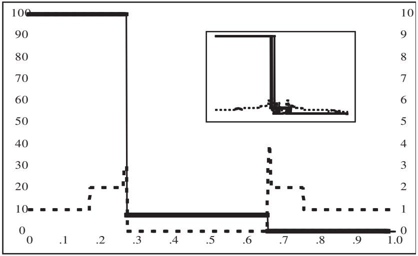
b

Myopic vs. Trend-Following: $n_1(0)$

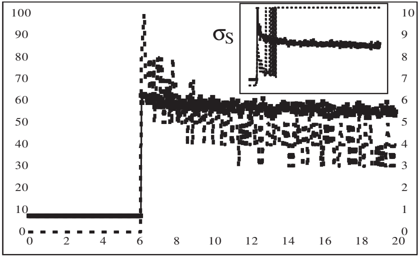


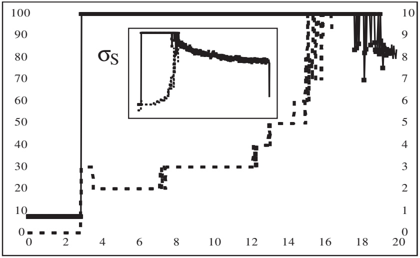
Myopic vs. Trend-Following: a_1





Myopic vs. Trend-Following: w_S





Control & Subsidies

- Can attain complete adoption
  - Minimize time of adoption, costly
  - Minimize required subsidy, takes time
- Determine societal cost of mixed innovations

Game of Scale Dynamics

- Trend-following usually pushes innovation faster
- Critical mass is important
- Can become stuck suboptimally

What If No Switching Costs?

- "Minority Game": Challet & Zhang
  - Bounded memory of history
  - Aggregate result public
  - Individual actions private
  - Models from spin glasses
  - Active research area since '97
- "Majority Game": Marsili
  - Apply minority game dynamics to reverse game
- Good intros: Esteban Moro '04, "Minority Games" by Challet et al.

Minority Game

- Inspired by Arthur's El Farol bar problem
- $N \gg 1$ agents
- Action: $a_i(t) \in \{-1, 1\}$
- $A(t) = \sum_{i=1}^{N} a_i(t)$
- Payoff: $-a_i(t) g\left(A(t)\right)$
  - g is odd
  - $g(x) = \text{sign}(x)$ or $g(x) = x/N$
- Public knowledge: $W(t+1) = \text{sign}\, A(t)$

Memories & Strategies

- Only remember last m results (bounded rationality)
- 2^m possible strategy sets to find $a_i(t)$
- Typed agents:
  - Endowed with set of strategies, function of m events
  - Evaluate each strategy after every round
  - Use strategy that has gained the most utility so far
  - Model of confirmation bias

Volatility & Information

- Notation:
  - $\overline{x}$: average over possible games
  - $\langle x \rangle_t$: average over long times
- Volatility
  - $\sigma^2 = \overline{\left\langle (A(t) - \langle A(t) \rangle_t)^2 \right\rangle_t}$
  - Smaller σ^2 means more winners
- Free/"Unused" information in history
  - $H = \frac{1}{2^m} \sum_{\nu=1}^{2^m} \langle W(t+1) | \text{history} = \nu \rangle_t^2$
  - Measures info content of series & asymmetry of response to available info

Volatility

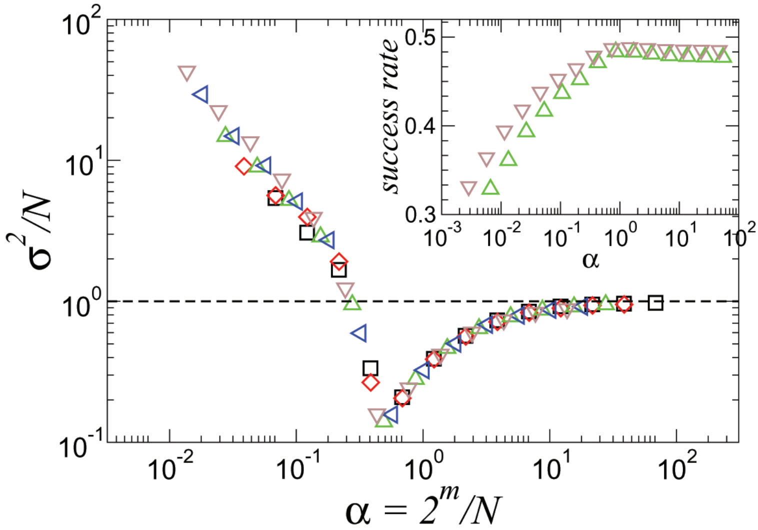


Figure from Esteban Moro, '04

Information & Frozen Agents

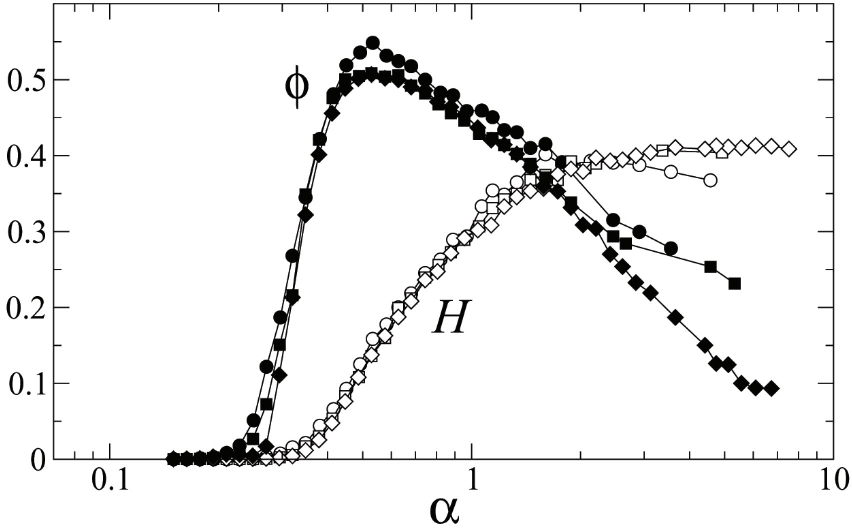
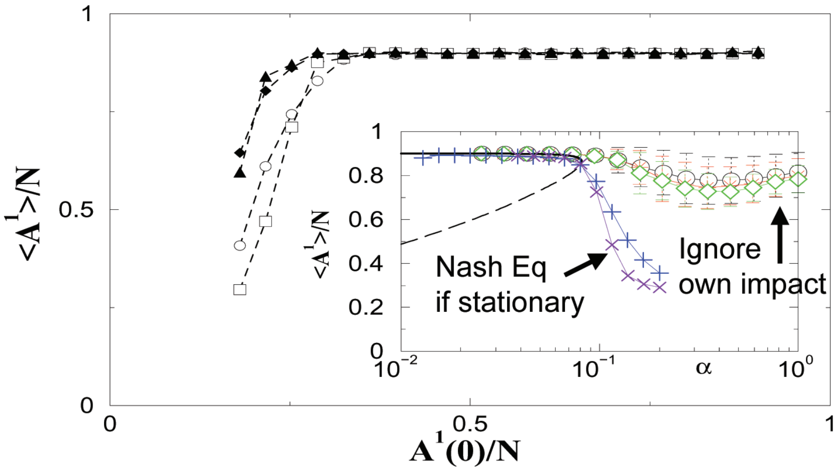


Figure from Esteban Moro, '04

Majority and Minority Game Themes

- Building up to markets
  - Trend followers (fundamentalists)
  - Contrarians
- Convergence (or lack thereof)
- Difficult to account for impact of own actions

Coordination of Best Strategies



A^1 = result given a particular history

Overlap: % agents with same outcome for same history

Figure from Kozłowski & Marsil, '03

Asymmetric Payoffs

- Dindo '04
- Replicator dynamics formulation
- Bifurcations in symmetric case
- Chaotic regions in asymmetric case (approx 2/3 of parameter space)

In Conclusion

- Economy of scale function secondary concern
- Switching costs very important
- Trend following (usually) good for coordination
- Asymmetry can slow/stop coordination